

# On $Q$ -Fuzzy Ideal Extensions in Semigroups

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## Abstract

In this paper the concept of extension of a  $Q$ -fuzzy ideal in semigroups has been introduced and some important properties have been studied.

**AMS Mathematics Subject Classification(2000):** 04A72, 20M12

**Key Words and Phrases:** Semigroup, Fuzzy set,  $Q$ -fuzzy set,  $Q$ -fuzzy ideal,  $Q$ -fuzzy completely prime( $Q$ -fuzzy completely semiprime) ideal,  $Q$ -fuzzy ideal extension.

## 1 Introduction

A semigroup is an algebraic structure consisting of a non-empty set  $S$  together with an associative binary operation[1]. The formal study of semigroups began in the early 20th century. Semigroups are important in many areas of mathematics, for example, coding and language theory, automata theory, combinatorics and mathematical analysis. The concept of fuzzy sets was introduced by *Lofti Zadeh*[12] in his classic paper in 1965. *Azirel Rosenfeld*[8] used the idea of fuzzy set to introduce the notions of fuzzy subgroups. *Nobuaki Kuroki*[4, 5, 6] is the pioneer of fuzzy ideal theory of semigroups. The idea of fuzzy subsemigroup was also introduced by *Kuroki*[4, 6]. In [5], *Kuroki* characterized several classes of semigroups in terms of fuzzy left, fuzzy right and fuzzy bi-ideals. Others who worked on fuzzy semigroup theory, such as *X.Y. Xie*[10, 11], *Y.B. Jun*[2], are mentioned in the bibliography. *X.Y. Xie*[10] introduced the idea of extensions of fuzzy ideals in semigroups. *K.H. Kim*[3] studied intuitionistic  $Q$ -fuzzy ideals in semigroups. In this paper the notion of extension of  $Q$ -fuzzy ideals in semigroups has been introduced and some important properties have been investigated.

## 2 Preliminaries

In this section we discuss some elementary definitions that we use in the sequel.

**Definition 2.1.** [7] If  $(S, *)$  is a mathematical system such that  $\forall a, b, c \in S, (a * b) * c = a * (b * c)$ , then  $*$  is called associative and  $(S, *)$  is called a *semigroup*.

**Definition 2.2.** [7] A *semigroup*  $(S, *)$  is said to be commutative if for all  $a, b \in S$ ,  $a * b = b * a$ .

**Definition 2.3.** [7] A semigroup  $S$  is said to be *left (right) regular* if, for each element  $a$  of  $S$ , there exists an element  $x$  in  $S$  such that  $a = xa^2$  (resp.  $a = a^2x$ ).

**Definition 2.4.** [7] A semigroup  $S$  is called *intra-regular* if for each element  $a$  of  $S$ , there exist elements  $x, y \in S$  such that  $a = xa^2y$ .

**Definition 2.5.** [7] A semigroup  $S$  is called *regular* if for each element  $a$  of  $S$ , there exists an element  $x \in S$  such that  $a = axa$ .

**Definition 2.6.** [7] A semigroup  $S$  is called *archimedean* if for all  $a, b \in S$ , there exists a positive integer  $n$  such that  $a^n \in SbS$ .

Throughout the paper unless otherwise stated  $S$  will denote a semigroup.

**Definition 2.7.** [7] A *subsemigroup* of a semigroup  $S$  is a non-empty subset  $I$  of  $S$  such that  $I^2 \subseteq I$ .

**Definition 2.8.** [7] A subsemigroup  $I$  of a semigroup  $S$  is called an *interior ideal* of  $S$  if  $SIS \subseteq I$ .

**Definition 2.9.** [7] A *left (right) ideal* of a semigroup  $S$  is a non-empty subset  $I$  of  $S$  such that  $SI \subseteq I$  ( $IS \subseteq I$ ). If  $I$  is both a left and a right ideal of a semigroup  $S$ , then we say that  $I$  is an *ideal* of  $S$ .

**Definition 2.10.** [7] Let  $S$  be a semigroup. Then an ideal  $I$  of  $S$  is said to be (i) *completely prime* if  $xy \in I$  implies that  $x \in I$  or  $y \in I \forall x, y \in S$ , (ii) *completely semiprime* if  $x^2 \in I$  implies that  $x \in I, \forall x \in S$ .

**Definition 2.11.** [12] A *fuzzy subset* of a non-empty set  $X$  is a function  $\mu : X \rightarrow [0, 1]$ .

**Definition 2.12.** [9] Let  $\mu$  be a fuzzy subset of a set  $X$ ,  $\alpha \in [0, 1 - \sup\{\mu(x) : x \in X\}]$ . Then  $\mu_\alpha^T$  is called a *fuzzy translation* of  $\mu$  if  $\mu_\alpha^T(x) = \mu(x) + \alpha \forall x \in X$ .

**Definition 2.13.** [9] Let  $\mu$  be a fuzzy subset of a set  $X$ ,  $\beta \in [0, 1]$ . Then  $\mu_\beta^M$  is called a *fuzzy multiplication* of  $\mu$  if  $\mu_\beta^M(x) = \beta \cdot \mu(x) \forall x \in X$ .

**Definition 2.14.** [9] Let  $\mu$  be a fuzzy subset of a set  $X$ ,  $\alpha \in [0, 1 - \sup\{\mu(x) : x \in X\}]$ , where  $\beta \in [0, 1]$ . Then  $\mu_{\beta\alpha}^C$  is called a *fuzzy magnified translation* of  $\mu$  if  $\mu_{\beta\alpha}^C(x) = \beta \cdot \mu(x) + \alpha \forall x \in X$ .

**Definition 2.15.** Let  $Q$  and  $X$  be two non-empty sets. A mapping  $\mu : X \times Q \rightarrow [0, 1]$  is called the *Q-fuzzy subset* of  $X$ .

**Definition 2.16.** Let  $\mu$  be a  $Q$ -fuzzy subset of a non-empty set  $X$ . Then the set  $\mu_t = \{x \in X : \mu(x, q) \geq t \forall q \in Q\}$  for  $t \in [0, 1]$ , is called the *level subset* or *t-level subset* of  $\mu$ .

EXAMPLE 1. Let  $S = \{a, b, c\}$  and  $*$  be a binary operation on  $S$  defined in the following caley table:

$*$	$a$	$b$	$c$
$a$	$a$	$a$	$a$
$b$	$b$	$b$	$b$
$c$	$c$	$c$	$c$

Then  $S$  is a semigroup. Let  $Q = \{p\}$ . Let us consider a  $Q$ -fuzzy subset  $\mu : S \times Q \rightarrow [0, 1]$ , by  $\mu(a, p) = 0.8, \mu(b, p) = 0.7, \mu(c, p) = 0.6$ . For  $t = 0.7$ ,  $\mu_t = \{a, b\}$ .

### 3 $Q$ -Fuzzy Ideals

**Definition 3.1.** A non-empty  $Q$ -fuzzy subset of a semigroup  $S$  is called a  $Q$ -fuzzy subsemigroup of  $S$  if  $\mu(xy, q) \geq \min\{\mu(x, q), \mu(y, q)\} \forall x, y \in S, \forall q \in Q$ .

**Definition 3.2.** A  $Q$ -fuzzy subsemigroup  $\mu$  of a semigroup  $S$  is called a  $Q$ -fuzzy interior ideal of  $S$  if  $\mu(xay, q) \geq \mu(a, q) \forall x, a, y \in S, \forall q \in Q$ .

**Definition 3.3.** A non-empty  $Q$ -fuzzy subset  $\mu$  of a semigroup  $S$  is called a  $Q$ -fuzzy left(right) ideal of  $S$  if  $\mu(xy, q) \geq \mu(y, q)$  (resp.  $\mu(xy, q) \geq \mu(x, q)$ )  $\forall x, y \in S, \forall q \in Q$ .

**Definition 3.4.** A non-empty  $Q$ -fuzzy subset  $\mu$  of a semigroup  $S$  is called a  $Q$ -fuzzy two-sided ideal or a  $Q$ -fuzzy ideal of  $S$  if it is both a  $Q$ -fuzzy left and a  $Q$ -fuzzy right ideal of  $S$ .

**Definition 3.5.** A  $Q$ -fuzzy ideal  $\mu$  of a semigroup  $S$  is called a  $Q$ -fuzzy completely prime ideal of  $S$  if  $\mu(xy, q) = \max\{\mu(x, q), \mu(y, q)\} \forall x, y \in S, \forall q \in Q$ .

**Definition 3.6.** A  $Q$ -fuzzy ideal  $\mu$  of a semigroup  $S$  is called a  $Q$ -fuzzy completely semiprime ideal of  $S$  if  $\mu(x, q) \geq \mu(x^2, q) \forall x \in S, \forall q \in Q$ .

**Theorem 3.7.** Let  $I$  be a non-empty subset of a semigroup  $S$  and  $\chi_{I \times Q}$  be the characteristic function of  $I \times Q$ , where  $Q$  is any non-empty set. Then  $I$  is a left ideal(right ideal, ideal, completely prime ideal, completely semiprime) of  $S$  if and only if  $\chi_{I \times Q}$  is a  $Q$ -fuzzy left ideal(resp.  $Q$ -fuzzy right ideal,  $Q$ -fuzzy ideal,  $Q$ -fuzzy completely prime ideal,  $Q$ -fuzzy completely semiprime ideal) of  $S$ .

*Proof.* Let  $I$  be a left ideal of a semigroup  $S$ . Let  $x, y \in S, q \in Q$ , then  $xy \in I$  if  $y \in I$ . It follows that  $\chi_{I \times Q}(xy, q) = 1 = \chi_{I \times Q}(y, q)$ . If  $y \notin I$ , then  $\chi_{I \times Q}(y, q) = 0$ . In this case  $\chi_{I \times Q}(xy, q) \geq 0 = \chi_{I \times Q}(y, q)$ . Therefore  $\chi_{I \times Q}$  is a  $Q$ -fuzzy left ideal of  $S$ .

Conversely, let  $\chi_{I \times Q}$  be a  $Q$ -fuzzy left ideal of  $S$ . Let  $x, y \in I, q \in Q$ , then  $\chi_{I \times Q}(x, q) = \chi_{I \times Q}(y, q) = 1$ . Now let  $x \in I$  and  $s \in S, q \in Q$ . Then  $\chi_{I \times Q}(sx, q) \geq \chi_{I \times Q}(x, q) = 1$ . Thus  $sx \in I$ . So  $I$  is a left ideal of  $S$ . Similarly we can prove that the other parts of the theorem.  $\square$

**Theorem 3.8.** Let  $S$  be a semigroup,  $Q$  be any non-empty set and  $\mu$  be a non-empty  $Q$ -fuzzy subset of  $S$ , then  $\mu$  is a  $Q$ -fuzzy left ideal( $Q$ -fuzzy right ideal,  $Q$ -fuzzy ideal,  $Q$ -fuzzy completely prime ideal,  $Q$ -fuzzy completely semiprime ideal) of  $S$  if and only if  $\mu_t$ 's

are left ideals (resp. right ideals, ideals, completely prime ideals, completely semiprime ideals) of  $S$  for all  $t \in \text{Im}(\mu)$ , where  $\mu_t = \{x \in S : \mu(x, q) \geq t \forall q \in Q\}$ .

*Proof.* Let  $\mu$  be a  $Q$ -fuzzy left ideal of  $S$ . Let  $t \in \text{Im} \mu$ , then there exist some  $\alpha \in S$  and  $q \in Q$  such that  $\mu(\alpha, q) = t$  and so  $\alpha \in \mu_t$ . Thus  $\mu_t \neq \phi$ . Again let  $s \in S$  and  $x \in \mu_t$ . Now  $\mu(sx, q) \geq \mu(x, q) \geq t$ . Therefore  $sx \in \mu_t$ . Thus  $\mu_t$  is a left ideal of  $S$ .

Conversely, let  $\mu_t$ 's are left ideals of  $S$  for all  $t \in \text{Im} \mu$ . Again let  $x, s \in S$  be such that  $\mu(x, q) = t \forall q \in Q$ . Then  $x \in \mu_t$ . Thus  $sx \in \mu_t$  (since  $\mu_t$  is a left ideal of  $S$ ). Therefore  $\mu(sx, q) \geq t = \mu(x, q)$ . Hence  $\mu$  is a  $Q$ -fuzzy left ideal of  $S$ . Similarly we can prove other parts of the theorem.  $\square$

## 4 $Q$ -Fuzzy Ideal Extensions

**Definition 4.1.** Let  $S$  be a semigroup,  $\mu$  be a  $Q$ -fuzzy subset of  $S$  and  $x \in S$ . The  $Q$ -fuzzy subset  $\langle x, \mu \rangle$  where  $\langle x, \mu \rangle : S \times Q \rightarrow [0, 1]$  is defined by  $\langle x, \mu \rangle (y, q) := \mu(xy, q) \forall y \in S, \forall q \in Q$  is called the  $Q$ -fuzzy extension of  $\mu$  by  $x$ .

EXAMPLE 2. Let  $X = \{1, \omega, \omega^2\}$  and  $Q = \{p\}$ . Let  $\mu$  be a  $Q$ -fuzzy subset of  $X$ , defined as follows

$$\mu(x, p) = \begin{cases} 0.3 & \text{if } x = 1 \\ 0.1 & \text{if } x = \omega \\ 0.5 & \text{if } x = \omega^2 \end{cases}.$$

Let  $x = \omega$ . Then the  $Q$ -fuzzy extension of  $\mu$  by  $\omega$  is given by

$$\langle x, \mu \rangle (y, p) = \begin{cases} 0.1 & \text{if } y = 1 \\ 0.5 & \text{if } y = \omega \\ 0.3 & \text{if } y = \omega^2 \end{cases}.$$

**Proposition 4.2.** Let  $S$  be a commutative semigroup,  $Q$  be any non-empty set and  $\mu$  be a  $Q$ -fuzzy ideal of  $S$ . Then  $\langle x, \mu \rangle$  is a  $Q$ -fuzzy ideal of  $S \forall x \in S$ .

*Proof.* Let  $\mu$  be a  $Q$ -fuzzy ideal of a commutative semigroup  $S$  and  $y, z \in S, q \in Q$ . Then

$$\langle x, \mu \rangle (yz, q) = \mu(xyz, q) \geq \mu(xy, q) = \langle x, \mu \rangle (y, q)$$

Hence  $\langle x, \mu \rangle$  is a  $Q$ -fuzzy right ideal of  $S$ . Hence  $S$  being commutative,  $\langle x, \mu \rangle$  is a  $Q$ -fuzzy ideal of  $S$ .  $\square$

REMARK 1. Commutativity of the semigroup  $S$  is not required to prove that  $\langle x, \mu \rangle$  is a  $Q$ -fuzzy right ideal of  $S$  when  $\mu$  is a  $Q$ -fuzzy right ideal of  $S$ .

**Definition 4.3.** Let  $S$  be a semigroup,  $Q$  be any non-empty set and  $\mu$  be a  $Q$ -fuzzy subset of  $S$ . Then we define  $\text{Supp } \mu = \{x \in S : \mu(x, q) > 0 \forall q \in Q\}$ .

**Proposition 4.4.** Let  $S$  be a semigroup,  $Q$  be any non-empty set,  $\mu$  be a  $Q$ -fuzzy ideal of  $S$  and  $x \in S$ . Then we have the following:

- (i)  $\mu \subseteq \langle x, \mu \rangle$ .
- (ii)  $\langle x^n, \mu \rangle \subseteq \langle x^{n+1}, \mu \rangle \forall n \in \mathbb{N}$ .
- (iii) If  $\mu(x, q) > 0$  then  $\text{Supp} \langle x, \mu \rangle = S \times Q$ .

*Proof.* (i) Let  $\mu$  be a  $Q$ -fuzzy ideal of  $S$  and let  $y \in S, q \in Q$ . Then  $\langle x, \mu \rangle (y, q) = \mu(xy, q) \geq \mu(y, q)$ . Hence  $\mu \subseteq \langle x, \mu \rangle$ .

(ii) Let  $\mu$  be a  $Q$ -fuzzy ideal of  $S$  and  $n \in \mathbb{N}, y \in S, q \in Q$ . Then  $\langle x^{n+1}, \mu \rangle (y, q) = \mu(x^{n+1}y, q) = \mu(xx^n y, q) \geq \mu(x^n y, q) = \langle x^n, \mu \rangle (y, q)$ . Hence  $\langle x^n, \mu \rangle \subseteq \langle x^{n+1}, \mu \rangle \forall n \in \mathbb{N}$ .

(iii) Since  $\langle x, \mu \rangle$  is a  $Q$ -fuzzy subset of  $S$ , so by definition,  $\text{Supp} \langle x, \mu \rangle \subseteq S \times Q$ . Let  $\mu$  be a  $Q$ -fuzzy ideal of  $S$  and  $y \in S, q \in Q$ . Then  $\langle x, \mu \rangle (y, q) = \mu(xy, q) \geq \mu(x, q) > 0$ . Consequently,  $(y, q) \in \text{Supp} \langle x, \mu \rangle$ . Thus  $S \times Q \subseteq \text{Supp} \langle x, \mu \rangle$ . Hence  $\text{Supp} \langle x, \mu \rangle = S \times Q$ .  $\square$

**Definition 4.5.** Let  $S$  be a semigroup,  $Q$  be any non-empty set,  $A \subseteq S$  and  $x \in S$ . We define  $\langle x, A \times Q \rangle = \{y \in S, q \in Q : (xy, q) \in A \times Q\}$ .

**Proposition 4.6.** Let  $S$  be a semigroup,  $Q$  be any non-empty set and  $\phi \neq A \subseteq S$ . Then  $\langle x, \mu_{A \times Q} \rangle = \mu_{\langle x, A \times Q \rangle}$  for every  $x \in S$ , where  $\mu_{A \times Q}$  denotes the characteristic function of  $A \times Q$ .

*Proof.* Let  $x, y \in S, q \in Q$ . Then two cases may arise viz. Case (i) :  $(y, q) \in \langle x, A \times Q \rangle$ . Case (ii) :  $(y, q) \notin \langle x, A \times Q \rangle$ .

Case (i) :  $(y, q) \in \langle x, A \times Q \rangle$ . Then  $(xy, q) \in A \times Q$ . This means that  $\mu_{A \times Q}(xy, q) = 1$  whence  $\langle x, \mu_{A \times Q} \rangle (y, q) = 1$ . Also  $\mu_{\langle x, A \times Q \rangle}(y, q) = 1$ .

Case (ii) :  $(y, q) \notin \langle x, A \times Q \rangle$ . Then  $(xy, q) \notin A \times Q$ . So  $\mu_{A \times Q}(xy, q) = 0$ . Thus  $\langle x, \mu_{A \times Q} \rangle (y, q) = 0$ . Again  $\mu_{\langle x, A \times Q \rangle}(y, q) = 0$ . Hence we conclude  $\langle x, \mu_{A \times Q} \rangle = \mu_{\langle x, A \times Q \rangle}$ .  $\square$

**Proposition 4.7.** Let  $S$  be a commutative semigroup,  $Q$  be any non-empty set and  $\mu$  be a  $Q$ -fuzzy completely prime ideal of  $S$ . Then  $\langle x, \mu \rangle$  is a  $Q$ -fuzzy completely prime ideal of  $S \forall x \in S$ .

*Proof.* Let  $\mu$  be a  $Q$ -fuzzy completely prime ideal of  $S$ . Then by Proposition 4.2,  $\langle x, \mu \rangle$  is a  $Q$ -fuzzy ideal of  $S$ . Let  $y, z \in S, q \in Q$ . Then

$$\begin{aligned}
 \langle x, \mu \rangle (yz, q) &= \mu(xyz, q) \text{ (cf. Definition 4.1)} = \max\{\mu(x, q), \mu(yz, q)\} \text{ (cf. Definition 3.5)} \\
 &= \max\{\mu(x, q), \max\{\mu(y, q), \mu(z, q)\}\} \\
 &= \max\{\max\{\mu(x, q), \mu(y, q)\}, \max\{\mu(x, q), \mu(z, q)\}\} \\
 &= \max\{\mu(xy, q), \mu(xz, q)\} \text{ (cf. Definition 3.5)} \\
 &= \max\{\langle x, \mu \rangle (y, q), \langle x, \mu \rangle (z, q)\} \text{ (cf. Definition 4.1)}
 \end{aligned}$$

Hence  $\langle x, \mu \rangle$  is a  $Q$ -fuzzy completely prime ideal of  $S$ .  $\square$

REMARK 2. Let  $S$  be a semigroup,  $Q$  be any non-empty set and  $\mu$  be a  $Q$ -fuzzy completely prime ideal of  $S$ . Then  $\langle x, \mu \rangle = \langle x^2, \mu \rangle$ .

**Proposition 4.8.** *Let  $S$  be a semigroup,  $Q$  be any non-empty set and  $\mu$  be a non-empty  $Q$ -fuzzy subset of  $S$ . Then for any  $t \in [0, 1]$ ,  $\langle x, \mu_t \rangle = \langle x, \mu \rangle_t \forall x \in S$ .*

*Proof.* Let  $y \in \langle x, \mu \rangle_t, q \in Q$ . Then  $\langle x, \mu \rangle(y, q) \geq t$ . This gives  $\mu(xy, q) \geq t$  and hence  $xy \in \mu_t$ . Consequently,  $y \in \langle x, \mu_t \rangle$ . It follows that  $\langle x, \mu \rangle_t \subseteq \langle x, \mu_t \rangle$ . Reversing the above argument we can deduce that  $\langle x, \mu_t \rangle \subseteq \langle x, \mu \rangle_t$ . Hence  $\langle x, \mu \rangle_t = \langle x, \mu_t \rangle$ .  $\square$

**Proposition 4.9.** *Let  $S$  be a commutative semigroup,  $Q$  be any non-empty set and  $\mu$  be a  $Q$ -fuzzy subset of  $S$  such that  $\langle x, \mu \rangle = \mu$  for every  $x \in S$ . Then  $\mu$  is a constant function.*

*Proof.* Let  $x, y \in S, q \in Q$ . Then by hypothesis we have  $\mu(x, q) = \langle y, \mu \rangle(x, q) = \mu(yx, q) = \mu(xy, q)$  (since  $S$  is commutative)  $= \langle x, \mu \rangle(y, q) = \mu(y, q)$ . Hence  $\mu$  is a constant function.  $\square$

**Corollary 4.10.** *Let  $S$  be a commutative semigroup,  $Q$  be any non-empty set and  $\mu$  be a  $Q$ -fuzzy completely prime ideal of  $S$ . If  $\mu$  is not constant,  $\mu$  is not a maximal  $Q$ -fuzzy completely prime ideal of  $S$ .*

*Proof.* Let  $\mu$  be a  $Q$ -fuzzy completely prime ideal of  $S$ . Then, by Proposition 4.7, for each  $x \in S$ ,  $\langle x, \mu \rangle$  is a  $Q$ -fuzzy completely prime ideal of  $S$ . Now by Proposition 4.4(i),  $\mu \subseteq \langle x, \mu \rangle$  for all  $x \in S$ . If  $\mu = \langle x, \mu \rangle$  for all  $x \in S$  then by Proposition 4.9,  $\mu$  is constant which is not the case by hypothesis. Hence there exists  $x \in S$  such that  $\mu \subsetneq \langle x, \mu \rangle$ . This completes the proof.  $\square$

**Proposition 4.11.** *Let  $S$  be a commutative semigroup,  $Q$  be any non-empty set and  $\mu$  be a  $Q$ -fuzzy completely semiprime ideal of  $S$ . Then  $\langle x, \mu \rangle$  is a  $Q$ -fuzzy completely semiprime ideal of  $S$  for all  $x \in S$ .*

*Proof.* Let  $\mu$  be a  $Q$ -fuzzy completely semiprime ideal of  $S$  and  $q \in Q$ . Then  $\mu$  is a  $Q$ -fuzzy ideal of  $S$  and hence by Proposition 4.2,  $\langle x, \mu \rangle$  is a  $Q$ -fuzzy ideal of  $S$ . Let  $y \in S$ . Then

$$\begin{aligned} \langle x, \mu \rangle(y^2, q) &= \mu(xy^2, q) \text{ (cf. Definition 4.1)} \leq \mu(xy^2x, q) \text{ (since } \mu \text{ is a } Q\text{-fuzzy ideal} \\ &\quad \text{ideal of } S) = \mu(xyyx, q) = \mu(xyxy, q) \text{ (since } S \text{ is commutative)} \\ &= \mu((xy)^2, q) \leq \mu(xy, q) \text{ (cf. Definition 3.6)} \\ &= \langle x, \mu \rangle(y, q) \text{ (cf. Definition 4.1)} \end{aligned}$$

Hence  $\langle x, \mu \rangle$  is a  $Q$ -fuzzy completely semiprime ideal of  $S$ .  $\square$

**Corollary 4.12.** Let  $S$  be a commutative semigroup,  $Q$  be any non-empty set and  $\{\mu_i\}_{i \in \Lambda}$  be a family of  $Q$ -fuzzy completely semiprime ideals of  $S$ . Let  $\lambda = \bigcap_{i \in \Lambda} \mu_i$ . Then for any  $x \in S$ ,  $\langle x, \lambda \rangle$  is a  $Q$ -fuzzy completely semiprime ideal of  $S$ , provided  $\lambda$  is non-empty.

*Proof.* Let  $x, y \in S, q \in Q$ . Then

$$\lambda(xy, q) = \inf_{i \in \Lambda} \mu_i(xy, q) \geq \inf_{i \in \Lambda} \mu_i(x, q) = \lambda(x, q)$$

Hence  $S$  being commutative semigroup,  $\lambda$  is a  $Q$ -fuzzy ideal of  $S$ .

Again let  $a \in S, q \in Q$ . Then

$$\lambda(a, q) = \inf_{i \in \Lambda} \mu_i(a, q) \geq \inf_{i \in \Lambda} \mu_i(a^2, q) = \lambda(a^2, q)$$

Consequently,  $\lambda = \bigcap_{i \in \Lambda} \mu_i$  is a  $Q$ -fuzzy completely semiprime ideal of  $S$ . Hence by Proposition 4.11,  $\langle x, \lambda \rangle$  is a  $Q$ -fuzzy completely semiprime ideal of  $S$ .  $\square$

REMARK 3. The proof of the above corollary shows that in a semigroup the non-empty intersection of family of  $Q$ -fuzzy completely semiprime ideals is a  $Q$ -fuzzy completely semiprime ideal.

**Corollary 4.13.** Let  $S$  be a commutative  $\Gamma$ -semigroup,  $Q$  be any non-empty set and  $\{S_i\}_{i \in I}$  a non-empty family of completely semiprime ideals of  $S$  and  $A := \bigcap_{i \in I} S_i \neq \phi$ . Then  $\langle x, \mu_A \rangle$  is a  $Q$ -fuzzy completely semiprime ideal of  $S$  for all  $x \in S$ , where  $\mu_A$  is the characteristic function of  $A$ .

*Proof.*  $A = \bigcap_{i \in I} S_i \neq \phi$  (by the given condition). Hence  $\mu_{A \times Q} \neq \phi$ . Let  $x \in S, q \in Q$ . Then  $x \in A$  or  $x \notin A$ . If  $(x, q) \in A \times Q$  then  $\mu_{A \times Q}(x, q) = 1$  and  $(x, q) \in S_i \times Q \forall i \in I$ . Hence

$$\inf\{\mu_{S_i \times Q} : i \in I\}(x, q) = \inf_{i \in I} \{\mu_{S_i \times Q}(x, q)\} = 1 = \mu_{A \times Q}(x, q).$$

If  $x \notin A$  then  $\mu_{A \times Q}(x, q) = 0$  and for some  $i \in I$ ,  $(x, q) \notin S_i \times Q$ . It follows that  $\mu_{S_i \times Q}(x, q) = 0$ . Hence

$$\inf\{\mu_{S_i \times Q} : i \in I\}(x, q) = \inf_{i \in I} \{\mu_{S_i \times Q}(x, q)\} = 0 = \mu_{A \times Q}(x, q).$$

Thus we see that  $\mu_{A \times Q} = \inf\{\mu_{S_i \times Q} : i \in I\}$ . Hence  $\mu_A = \inf\{\mu_{S_i} : i \in I\}$  (cf. Corollary 4.12). Again  $\mu_{S_i}$  is a  $Q$ -fuzzy completely semiprime ideal of  $S$  for all  $i \in I$ . Consequently by Corollary 4.13, for all  $x \in S$ ,  $\langle x, \mu_A \rangle$  is a  $Q$ -fuzzy completely semiprime ideal of  $S$ .  $\square$

We can obtain following results by routine verification.

**Theorem 4.14.** Let  $S$  be a semigroup,  $Q$  be any non-empty set and  $\mu$  be a  $Q$ -fuzzy completely prime ideal of  $S$ . Then  $\langle x, \mu_{\beta_\alpha}^C \rangle$  is a  $Q$ -fuzzy completely prime ideal of  $S$ .

**Theorem 4.15.** *Let  $S$  be a semigroup,  $Q$  be any non-empty set and  $\mu$  be a  $Q$ -fuzzy right ideal of  $S$ . Then  $\langle x, \mu_{\beta\alpha}^C \rangle$  is a  $Q$ -fuzzy right ideal of  $S$ .*

**Theorem 4.16.** *Let  $S$  be a commutative semigroup,  $Q$  be any non-empty set and  $\mu$  be a  $Q$ -fuzzy ideal of  $S$ . Then  $\langle x, \mu_{\beta\alpha}^C \rangle$  is a  $Q$ -fuzzy ideal of  $S$ .*

**Theorem 4.17.** *Let  $S$  be a commutative semigroup and  $Q$  be any non-empty set and  $\mu$  be a  $Q$ -fuzzy completely semiprime ideal of  $S$ . Then  $\langle x, \mu_{\beta\alpha}^C \rangle$  is a  $Q$ -fuzzy completely semiprime ideal of  $S$ .*

**Theorem 4.18.** *Let  $S$  be a commutative semigroup and  $Q$  be any non-empty set and  $\mu$  be a  $Q$ -fuzzy interior ideal of  $S$ . Then  $\langle x, \mu_{\beta\alpha}^C \rangle$  is a  $Q$ -fuzzy interior ideal of  $S$ .*

**Theorem 4.19.** *Let  $S$  be a regular commutative semigroup,  $Q$  be any non-empty set and  $\mu$  be a  $Q$ -fuzzy ideal of  $S$ . Then  $\langle x, \mu_{\beta\alpha}^C \rangle$  is a  $Q$ -fuzzy completely semiprime ideal of  $S$ .*

**Theorem 4.20.** *Let  $S$  be a right regular semigroup,  $Q$  be any non-empty set and  $\mu$  be a  $Q$ -fuzzy right ideal of  $S$ . Then  $\langle x, \mu_{\beta\alpha}^C \rangle$  is a  $Q$ -fuzzy completely semiprime right ideal of  $S$ .*

**Theorem 4.21.** *Let  $S$  be an intra-regular commutative semigroup,  $Q$  be any non-empty set and  $\mu$  be a  $Q$ -fuzzy ideal of  $S$ . Then  $\langle x, \mu_{\beta\alpha}^C \rangle$  is a  $Q$ -fuzzy completely semiprime ideal of  $S$ .*

**Theorem 4.22.** *Let  $S$  be an archimedean commutative semigroup,  $Q$  be any non-empty set and  $\mu$  be a  $Q$ -fuzzy completely semiprime ideal  $\langle x, \mu_{\beta\alpha}^C \rangle$  of  $S$  is a constant function.*

REMARK 4. If we put  $\beta = 1$ (respectively  $\alpha = 0$ ) in fuzzy magnified translation then it reduces to fuzzy translation(respectively fuzzy multiplication). Consequently analogues of Theorems 4.14-4.22 follow easily in fuzzy translation and fuzzy multiplication.

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